

Unsupervised Speech Enhancement with Diffusion-based Generative Models

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What is speech enhancement?

- In practice, speech is recorded in noisy environments → **speech enhancement** (SE)



SE: Given **noisy speech** observation $\mathbf{x} = \mathbf{s} + \mathbf{n}$, estimate the **clean speech** signal \mathbf{s} .

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- Complex-valued short-time Fourier transform domain

Approaches to SE

- ▷ **Supervised:** Model $p_{\theta}(\mathbf{s}|\mathbf{x})$

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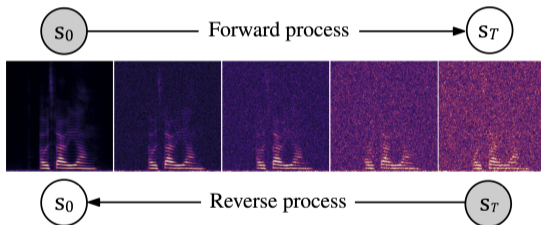
- **Training** - Learn speech's prior distribution $p_{\theta}(\mathbf{s})$ - *only* on **clean** speech signals
- **Inference** - Model $p_{\phi}(\mathbf{x}|\mathbf{s})$, and infer \mathbf{s} using $p_{\theta}(\mathbf{s})$

☞ May offer superior generalization

Score-based generative models for SE

▷ **Previous (supervised) diffusion-based work: SMGSE+**¹

- Gradually corrupt clean speech with both Gaussian and environmental noise

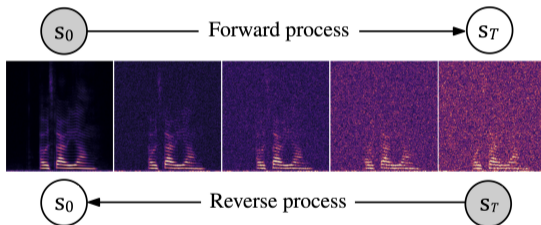


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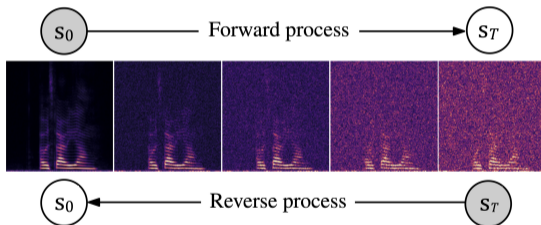


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- Processes can be modelled as a *Stochastic Differential Equation (SDE)*

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$$p_{\phi}(\mathbf{x}|\mathbf{s}) = \mathcal{N}_{\mathbb{C}}(\mathbf{s}, \text{diag}(\mathbf{v}_{\phi}))$$

- $\mathbf{v}_{\phi} = \text{vec}(\mathbf{WH}) \leftarrow$ non-negative matrix factorisation (NMF)

Inference framework: Expectation-maximisation

▷ Iterative **Expectation Maximisation**-based inference ($k = 1, \dots, K$):

1. **E-step:** *Draw posterior sample*

$$\hat{\mathbf{s}}_k \sim p_{\Theta_{k-1}}(\mathbf{x}|\mathbf{s}) \quad \rightarrow \text{reverse diffusion}$$

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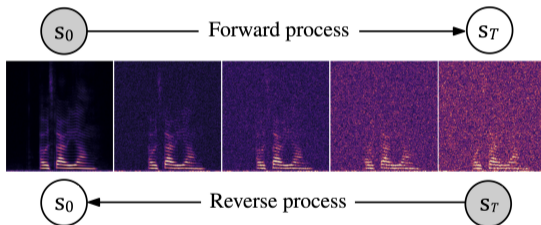
2. **M-step:** *Maximise likelihood*

$$\phi_k \leftarrow \underset{\phi}{\operatorname{argmax}} \log p_{\phi}(\mathbf{x}|\hat{\mathbf{s}}_k) \quad \rightarrow \text{NMF update}$$

Prior: Diffusion-based speech generative model

▷ *Unconditional* (prior) diffusion model for complex-valued **clean speech** STFT:

- **Noising (forward) SDE:**² $ds_t = \mathbf{f}(s_t)dt + g(t)d\mathbf{w}$, $\mathbf{f}(s_t) = -\gamma s_t$

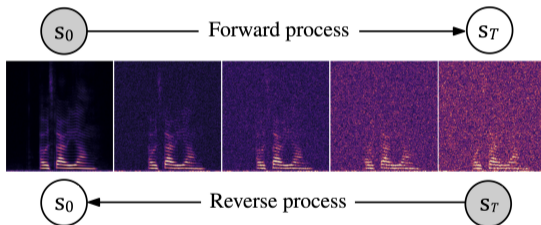


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Prior: Approximating the score

Knowing the score function enables sampling from the prior. Approximate it instead:

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$$\theta^* = \operatorname{argmin}_{\theta} \mathbb{E}_{t, \mathbf{s}, \zeta, \mathbf{s}_t | \mathbf{s}} \left[\left\| \mathbf{S}_{\theta}(\mathbf{s}_t, t) + \frac{\zeta}{\sigma(t)} \right\|_2^2 \right], \quad \zeta \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$$

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2. Numerically sample from the prior $p_{\theta}(\mathbf{s})$

☞ The above SDE can be solved by the *Predictor-Corrector (PC) sampler*

SE phase as EM approach

Once the **prior score model** is trained, SE is performed via EM:

E-step: Approximate the conditional reverse SDE:

$$d\mathbf{s}_t = \left[\mathbf{f}(\mathbf{s}_t) - g(t)^2 \nabla_{\mathbf{s}_t} \log p_t(\mathbf{s}_t | \mathbf{x}) \right] dt + g(t) d\mathbf{w}$$

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Approximation by the “*noise-perturbed pseudo-likelihood score*”³ $\nabla_{\mathbf{s}_t} \log \tilde{p}_\phi(\mathbf{x} | \mathbf{s}_t)$

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- λ : weighting parameter to balance prior and likelihood terms.

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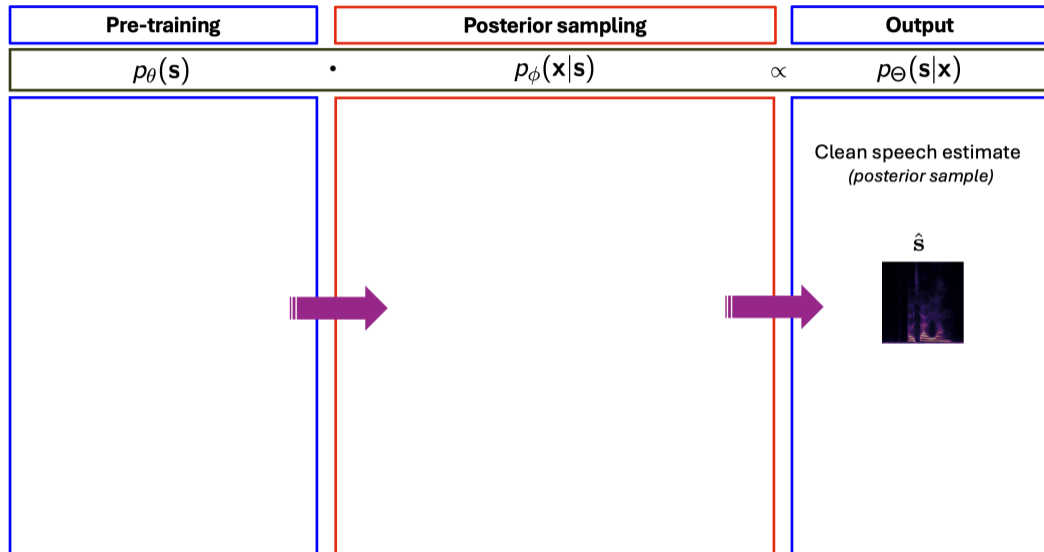
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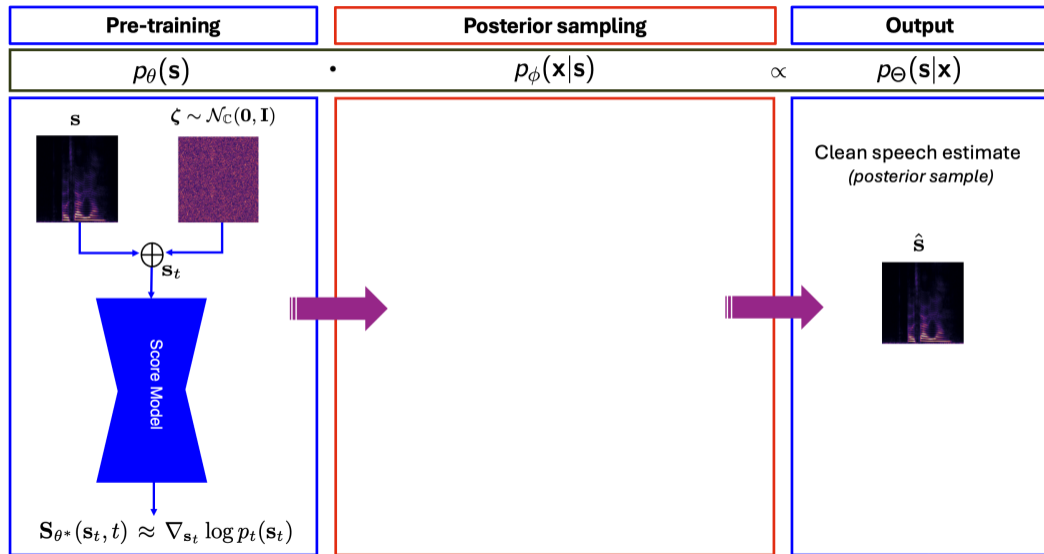
M-step:

$$\begin{aligned} \phi^* &\leftarrow \operatorname{argmax}_{\mathbf{v}_\phi(i) \geq 0} \log p_\phi(\mathbf{x}|\hat{\mathbf{s}}) \\ &= \operatorname{argmin}_{\mathbf{v}_\phi(i) \geq 0} \sum_i \frac{(\mathbf{x} - \hat{\mathbf{s}})_i^* (\mathbf{x} - \hat{\mathbf{s}})_i}{\mathbf{v}_\phi(i)} + \log(\mathbf{v}_\phi(i)) \end{aligned}$$

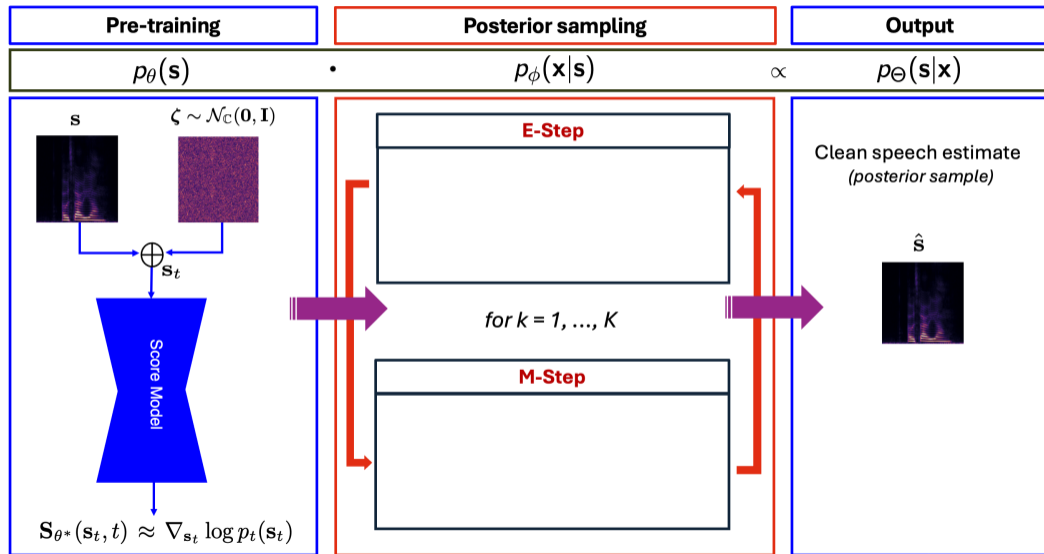
UDiffSE pipeline



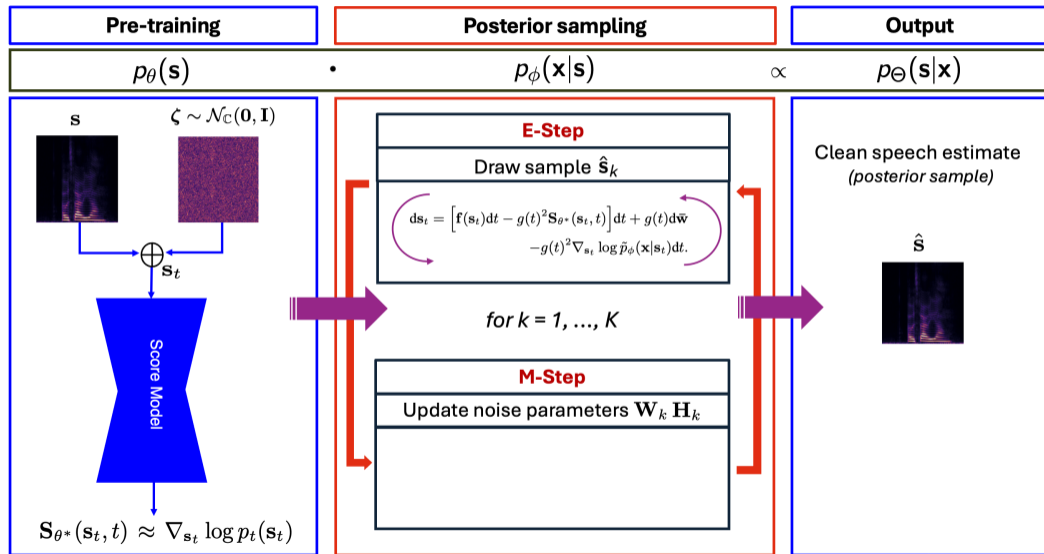
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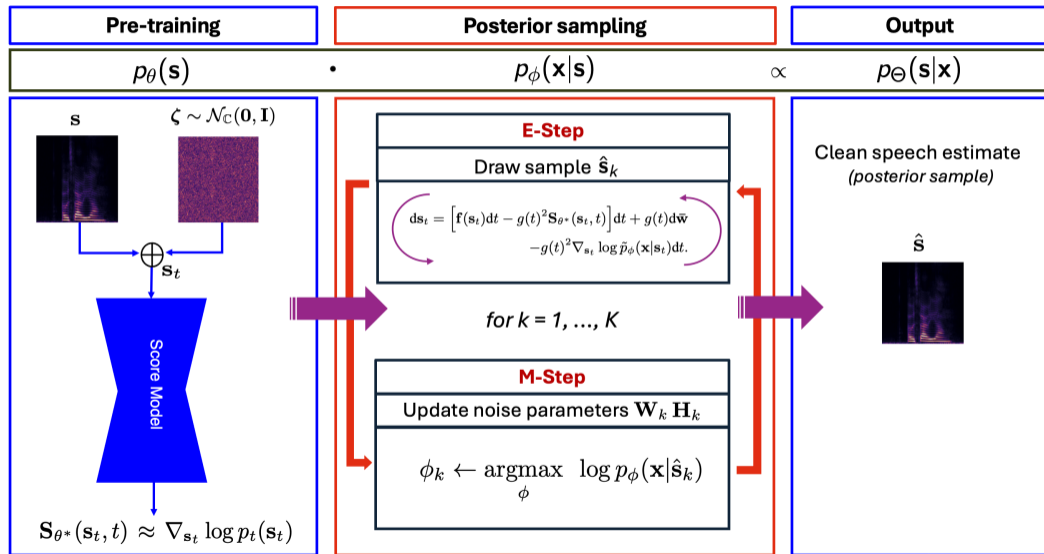
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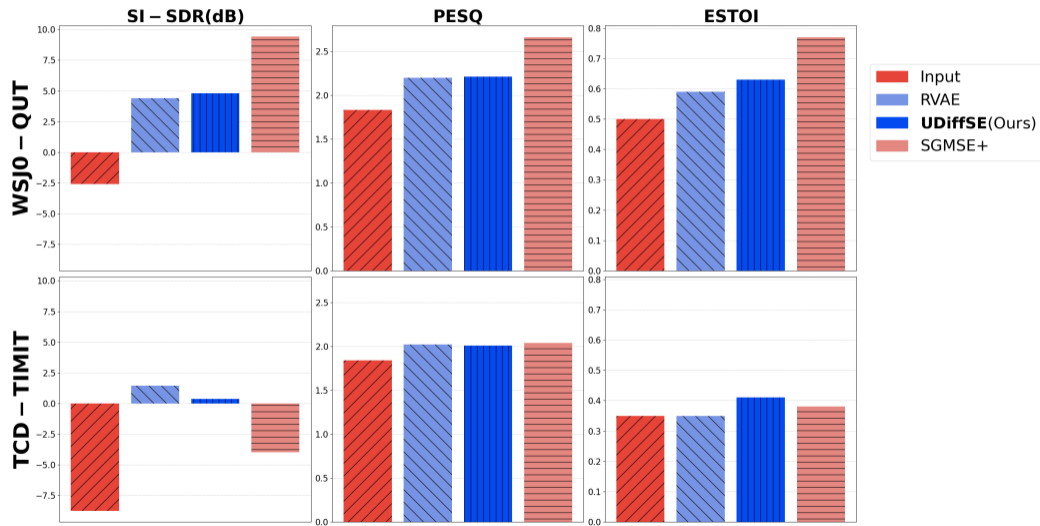


UDiffSE pipeline



- **Datasets.**
 - Training: WSJ0 (~ 25 hrs)
 - Testing: WSJ0-QUT (1.5hrs), TCD-TIMIT (45mins)
 - Noise levels (dB): $[-5, 0, 5]$.
 - Noise types: *Café*, *Home*, *Street*, and *Car*
- **Evaluation Metrics.**
 - Objective measures: SI-SDR, ESTOI, PESQ
 - (Pseudo)-subjective measures: DNS-MOS (SIG, BAK, OVRL)
- **Baselines.** RVAE, SGMSE+ (pre-trained).
- **Models architecture.** Multi-resolution U-Net as in SGMSE+.
- **EM settings.** NMF rank 4. $K = 5$ EM iterations. Averaging over $b = 4$ parallel sample batches. Weighting parameter $\lambda = 1.5$.

Results



Conclusion & next directions

▷ Conclusions

- UDiffSE: *Proof of concept*
- Learning an *implicit prior* distribution over clean speech data
- An EM approach to generate clean speech & learn the noise parameters *at the same time*
- Better *generalisation* & outperforms VAE (also less artifacts)

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▷ Next steps

1. Speeding up inference
2. Investigating generalisational capability
3. Improving prior

Further resources



GitHub

<https://github.com/joanne-b-nortier/udiffse>

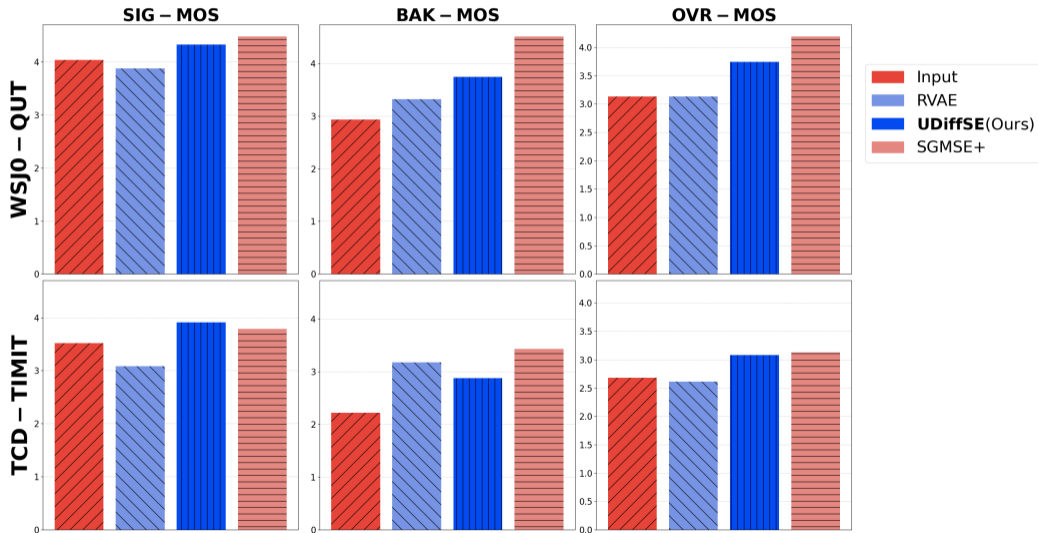


Demo

<https://team.inria.fr/multispeech/demos/udiffse/>

Additional resources

Results II



Algorithm 2 Posterior sampling (E-step) of UDiffSE

Require: \mathbf{x} , N , ℓ , λ , r (signal-to-noise ratio)

- 1: $\mathbf{s}_1 \sim \mathcal{N}_{\mathbb{C}}(\mathbf{x}, \mathbf{I})$, $\Delta\tau \leftarrow \frac{1}{N}$
 - 2: **for** $i = N, \dots, 1$ **do**
 - 3: $\tau \leftarrow \frac{i}{N}$
 - 4: $\epsilon_\tau \leftarrow (\sigma_\tau \cdot r)^2$
 - 5: $\zeta_c \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$ \triangleright (Corrector)
 - 6: $\mathbf{s}_\tau \leftarrow \mathbf{s}_\tau + \epsilon_\tau \mathbf{S}_{\theta^*}(\mathbf{s}_\tau, \tau) + \sqrt{2\epsilon_\tau} \zeta_c$
 - 7: $\zeta_p \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$ \triangleright (Predictor)
 - 8: $\mathbf{s}_\tau \leftarrow \mathbf{s}_\tau - \mathbf{f}_\tau \Delta\tau + g_\tau^2 \mathbf{S}_{\theta^*}(\mathbf{s}_\tau, \tau) \Delta\tau + g_\tau \sqrt{\Delta\tau} \zeta_p$
 - 9: **if** $i \equiv 0 \pmod{\ell}$ **then** \triangleright (Posterior)
 - 10: $\nabla_{\mathbf{s}_\tau} \log \tilde{p}_\phi(\mathbf{x}|\mathbf{s}_\tau) \leftarrow \frac{1}{\delta_\tau} \left[\frac{\sigma_\tau^2}{\delta_\tau^2} \mathbf{I} + \text{diag}(\mathbf{v}_\phi) \right]^{-1} \left(\frac{\mathbf{s}_\tau}{\delta_\tau} - \mathbf{x} \right)$
 - 11: $\mathbf{s}_\tau \leftarrow \mathbf{s}_\tau + \lambda g_\tau^2 \nabla_{\mathbf{s}_\tau} \log \tilde{p}_\phi(\mathbf{x}|\mathbf{s}_\tau) \Delta\tau$
 - 12: **end if**
 - 13: **end for**
 - 14: **return** $\hat{\mathbf{s}} = \mathbf{s}_0$
-